MULTIPLE-COINCIDENCE OF FLOOD WAVES ON THE MAIN RIVER AND ITS TRIBUTARIES

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Abstract

This paper addresses the definition of multiple coincidences of flood waves on the main river and its tributaries. Contrary to previous studies of partial coincidences of various flood parameters (Prohaska, 1999) for the main river and one of its tributaries, this procedure allows for the definition of complex (multiple) coincidences of a single parameter for the main river and several of its tributaries. In particular, coincidence is defined for the major parameter which characterizes a flood (i.e., the flood wave volume).

The paper gives a practical example of analysis of simultaneous flood waves on the Danube and its main tributaries in Serbia: the Tisa and the Sava Rivers. The procedure for potential use of the established coincidence functions in applied water management and forecasting is also described in the paper.

Keywords: Multiple-coincidence, flood wave, flood wave volume, two-dimensional distribution function, cumulative probability function, exceeding cumulative probability.

INTRODUCTORY REMARKS

Development of flood protection systems requires comprehensive information on flood characteristics. Design of engineering structures, i.e. determining of their capacities, requires facts on peak discharge. In addition to the maximum discharge, analyses and evaluation of effects of reservoirs and/or river channel on flood wave transformation involve data on flood wave volume and shape of the hydrograph. Therefore, safety of the protected area/structure, or the risk to which it is exposed, can be related to exceedance probability of appropriate flood characteristics.

Classical approach to evaluation of flood risk leads to evaluation of the probability that a predefined magnitude of relevant flood characteristic will be exceeded. This is equivalent to a determination of return period of the considered event. Realization of this task involves statistical analysis of hydrologic time series at adjacent gauging station. This approach gives satisfactory results in solving many engineering problems, particularly in dealing with flood protection of simple cases, such as flood protection along the river reach without significant tributary on it.

However, when the river receives a significant tributary, above mentioned approach does not yield reliable estimates of flood properties. Namely, flood concentration on two
merging rivers is usually different, so that the floods on them do not appear simultaneously. Furthermore, flood wave occurring at one river, my significantly affect flow pattern in the other stream. Yet, the regular processing of hydrologic time series cannot assess the impact of one river upon another, for the data are commonly collected at gauging stations located beyond area of influence. Under the circumstances, it is often important to assess coincidence of flood waves in the main river and its tributary. In a case when there are more significant tributaries, the problem becomes even more complex and there is a need for a comprehensive consideration of hydraulic and hydrological conditions within a broader region. Random variables that describe the hydrological processes should be viewed as multidimensional event in order to fully determine the risk which is the most frequently adopted criterion for design of flood protection structures and/or systems. Application of two-dimensional or even multidimensional approach to hydrological random variables, which is proposed in this paper, can help in overcoming the encountered difficulties.

METHODOLOGY FOR EVALUATION OF COINCIDENCE OF MULTI-DIMENSIONAL FLOOD WAVES

The term coincidence is used to denote simultaneous occurrence of floods at two (or more) rivers. A degree of coincidence is measured by the probability of the events. Theoretical background is founded upon practical application of multivariate probability distribution function, or more precisely, by their conditional probabilities. In this particular case one considers quantitative characteristics of two simultaneous flood waves on the main river and its tributary. Usually, subjects of analysis are: maximum discharge of the flood hydrograph, volume of the flood, duration of the flood or time between the maximum flows during the analyzed flood.

Procedure for calculation of multidimensional coincidence of flood flows on complex river systems stems from the developed methodology for evaluation of coincidence of floods at two adjacent streams (Prohaska, 1999). The coincidence is equivalent to the probability that two variables, X and Y, will occur simultaneously, where X and Y represent random event (flood wave characteristics) at the considered river.

According to the theory of statistics, a two-dimensional probability distribution function of the normally distributed bi-variate random process, X and Y is defined as follows:

\[
f(x, y) = \frac{1}{2\pi \cdot \sigma_x \cdot \sigma_y \cdot \sqrt{1 - \rho^2}} \cdot \exp \left\{ - \frac{1}{2(1 - \rho^2)} \left[ \frac{(x - \mu_x)^2}{\sigma_x^2} - \frac{2\rho(x - \mu_x)(y - \mu_y)}{\sigma_x \cdot \sigma_y} + \frac{(y - \mu_y)^2}{\sigma_y^2} \right] \right\}
\]

(1)

The symbols in expressions (1) stand for the following:

- \(X\) and \(Y\) – random variables (flood flow characteristics at the main river and its tributary or at two adjacent profiles at the main river);
- \(x\) and \(y\) – simultaneous realization of the random variables \(X\) and \(Y\) respectively;
\( \mu_x \) and \( \mu_y \) – the expected values of \( X \) and \( Y \) variables;
\( \sigma_y \) and \( \sigma_x \) – the standard deviations of \( X \) and \( Y \) variables;
\( \rho \) – the coefficient of correlation between \( X \) and \( Y \) variables.

For a joint probability density function - j.p.d.f., \( f(x, y) \), the marginal densities, \( f(\cdot, y) \) and \( f(x, \cdot) \), are defined by:

\[
f(x, \cdot) = \int_{y=-\infty}^{y=y} f(x,y) \, dy
\]

(2)

\[
f(\cdot, y) = \int_{x=-\infty}^{x=x} f(x,y) \, dx
\]

(3)

The marginal cumulative probability functions are determined from:

\[
F(x, \cdot) = \int_{t=x}^{t=\infty} f(t, \cdot) \, dt
\]

(4)

And

\[
F(\cdot, y) = \int_{z=y}^{z=\infty} f(\cdot, z) \, dz
\]

(5)

The cumulative probability function - c.d.f., \( F(x,y) \), is evaluated from:

\[
F(x,y) = P[X \leq x \cap Y \leq y] = \int_{t=x}^{t=\infty} \int_{z=y}^{z=\infty} f(t,z) \, dt \, dz
\]

(6)

The exceedance cumulative probability, \( \Phi(x,y) \), can be obtained from the following relation:

\[
\Phi(x,y) = \int_{t=x}^{t=\infty} \int_{z=y}^{z=\infty} f(t,z) \, dt \, dz = P[X > x \cap Y > y] = 1 - P[X < x \cup Y < y] = 1 - F(x, \cdot) - F(\cdot, y) + F(x,y)
\]

(7)

In bi-variate statistical analyses of flood characteristics, hydrologists encounter two basic obstacles which must be resolved in a practical implementation of the proposed model. The first obstacle comes out from the fact that the most flood characteristics are not normally distributed. It is, however, customarily assumed that the considered variables follow the log-normal distribution. Therefore, their logarithmic transformations:

\[
U = \log X
\]

(8)

\[
W = \log Y
\]

(9)

are said to be normally distributed.

The previously described model appears to be direct method for evaluation of the cumulative distribution functions. Nevertheless, it can be lengthy and tiresome, for it
involves extensive calculation in a three-dimensional space X, Y, and \( \rho \). For the presently available computers it does not necessarily represent a severe obstacle. However, at the time when this project was first conceived, the method of direct computation was almost infeasible. For that reason, a more convenient graphoanalitical scheme was implemented. This scheme has been described by Abramowitz, M., & Stegun, A., (1972), which is briefly discussed in the ensuing text.

The scheme is dealing with the standard normal variables. A transformation of non-standard variables into the standardized ones, can be done by the well known procedure, namely:

\[
\psi = \frac{(u - \bar{U})}{\sigma_u} \quad (10)
\]

\[
\xi = \frac{(w - \bar{W})}{\sigma_w} \quad (11)
\]

The variables \( \psi \) and \( \xi \) are, according to the above stated assumption, normally distributed with the expected values \( \mu_\psi = \mu_\xi = 0 \), and the standard deviations \( \sigma_\psi = \sigma_\xi = 1 \).

With the above transformations, the joint probability density function can be written to read:

\[
f(\psi, \xi) = \frac{1}{2\pi \sqrt{1 - \rho^2}} \exp \left\{ \frac{1}{2(1 - \rho^2)} \left[ \psi^2 - 2\rho \psi \xi + \xi^2 \right] \right\} \quad (12)
\]

The values of the correlation coefficient \( \rho \) should be replace by \( R \), which can be calculated from the observed data using the standardized series With this parameter, and after simplifying notation in Eq.12, the following relation can be written:

\[
\iint_A f(\psi, \xi) d\psi d\xi = 1 - \exp \left\{ -\frac{\lambda^2}{2(1 - \rho^2)} \right\} \quad (13)
\]

The integral given by Eq. 13 over an area \( A \), i.e. the integral over the space \( \psi, \xi \in A \), namely represents the probability that the realization of events \( \psi = h \) and \( \xi = k \) will fall into the area \( A \) which is contoured by an ellipse given by the following equation

\[
\psi^2 - 2\rho \psi \xi + \xi^2 = \lambda^2 \quad (14)
\]

The newly introduced symbol \( \lambda \) obviously is related to the constant value of the integral (13). Consequently, it is related to the variables \( \psi \) and \( \xi \), as well as to the correlation coefficient.
Hence, for each value of \( \lambda = \text{const} \), the probability contained within the ellipse of Eq. 14 can be calculated.

\[
\xi^2 - 2\rho \psi \xi + (\psi^2 - \lambda^2) = 0
\]

(15)

As previously stated, any particular value of \( \lambda = \text{const} \) corresponds to an ellipse. Furthermore, any given value, \( \psi = h \), intersects the ellipse at two different values of \( \xi \), let say \( \xi = k_1 \) and \( \xi = k_2 \).

Hence, solving the quadratic equation (16) for any particular value of \( \lambda = \text{const} \) corresponding to the required level of probability given by Eq. (13), one obtains two particular coordinates (values of \( \xi \), let say \( \xi = k_1 \) and \( \xi = k_2 \)) representing intersection of the ellipse and the straight line \( \psi = h_0 \). Repeating the calculation for several selected values of \( \lambda \) while changing the values of \( \psi = h_0 \), a series of the ellipses can be constructed. It should be kept in mind that after each calculation, an appropriate transformation should be performed in correspondence with Eq. (11) and Eq. (10) to obtain non-standardized, natural values of the flood characteristics instead of standardized logarithmic values.

The described computational scheme is a rather direct. However, the results are not of a great use, except to give the analyst a general insight into the relation of the considered flood characteristics.

When it comes to evaluation of the cumulative distribution function, the direct method, as previously outlined, is not convenient. To overcome computational difficulties, the Abramowitz & Stegun's procedure was implemented in this study. The computational scheme uses a grapho-analytical procedure which defines the cumulative probability, \( \Phi(h,k,\rho) \), in terms of the probability \( \Phi(h,0,r) \) and \( \Phi(k,0,r) \), where instead of the correlation coefficient, \( \rho \), the value \( r = r(h,k,\rho) \), is used. The value \( r \) is related to \( h \) and \( k \), as well as to \( \rho \) itself. More specifically, the probability \( \Phi(h,k,\rho) \) can be evaluated from:

\[
\Phi(h,k,\rho) = \Phi\left(h,0,\frac{(ph-k) \cdot sgnh}{\sqrt{h^2-2phk+k^2}}\right) + \Phi\left(k,0,\frac{(pk-h) \cdot sgnk}{\sqrt{h^2-2phk+k^2}}\right) \begin{cases} 0 & \text{if } hk \geq 0 \text{ and } h+k \geq 0 \\ 1 & \text{for all other cases} \end{cases}
\]

(16)

where \( (sgnh) \) and \( (sgnk) \) are equal to 1 when \( h \) or \( k \), respectively are greater or equal to zero, while they become -1 whenever \( h \) and \( k \) are smaller than zero.

Further elaboration of the theory of multiple flood coincidence in a complex river system is adjusted to a river reach where the main river receives two significant tributaries and negligible lateral inflow. Flood volume was selected to represent the flood hydrograph. The probability distribution functions of flood volumes represent starting point in assessing a multidimensional probabilistic dependency of flood wave of the main river...
(at the downstream profile) as a function of floods at all the considered entering profiles
(of the main river and its tributary). It can be written:

$$P(W \geq w) = p$$

(17)

Where

$P(W \geq w)$ – probability that flood volume at all inflow/outflow profiles will be exceeded

$p$ – exceedance probability

In addition, the exceedance probability of simple coincidence between two inflow
profiles, $j$ and $k$, is required:

$$P((W_j \geq w_j) \cap (W_k \geq w_k)) = q$$

(18)

Where

$P((W_j \geq w_j) \cap (W_k \geq w_k))$ – exceeding probability of the coinciding flood volumes at two
considered inflowing profiles, $j$ and $k$

$q$ – exceedance probability

The procedure for setting the needed relationship consists of a step-by-step balancing of
the inflowing flood-volumes which correspond to different probabilities according to the
following balance equation:

$$W_{iz,p=\theta} = W_{i,p=T} + W_{j,r} + W_{k,z}$$

(19)

Where

$W_{iz,p=\theta}$ – flood volume at the exiting profile for a fixed probability $p=\theta$,

$W_{i,p=T}$ – flood volume at the $i$-th inflow profile for a selected return period $T$,

$W_{j,r}$ – flood volume at the $j$-th inflow profile for an arbitrary chosen
probability $r$, in accordance with exceedance probability $P((W_j \geq w_j) \cap (W_k \geq w_k))$,

$W_{k,z}$ – flood volume at the $k$-th inflow profile for an arbitrarily chosen
probability $z$, in accordance with exceedance probability $P((W_k \geq w_k) \cap (W_k \geq w_k))$.

For a fixed exceedance probability of the outflow hydrograph, $\theta$, and a selected return
period, $T$, using Eq. 17, theoretical values of flood volume, $W_{iz,p=\theta}$ and $W_{i,p=T}$, can be
determined. Arbitrarily chosen probability $r$ or $z$, let say $r$, allows for calculation of
theoretical values of $W_{j,r}$ and $W_{k,z}$, under condition that Eq. (18) (i.e. $P((W_j \geq w_j) \cap (W_k \geq w_k)) = q$) is satisfied. The corresponding probability, $W_{k,z}$, can also be obtained from Eq. (17).

In that way all values of corresponding probabilities can be defined: $p(W_{iz,p=\theta})$, $p(W_{i,p=T})$, $p(W_{j,r})$ and $p(W_{k,z})$. From this, a multidimensional relationship of the exceedance
probabilities of flood-volumes at the main river and its tributaries can be produced in the
following form:

$$p(W_{iz,p=\theta}) = P((p(W_{i,p=T}) \cap p(W_{j,r}) \cap p(W_{k,z}))$$

(20)
Where:
\[ p(W_{iz,p=\theta}) \] - Exceedance probability of the flood-wave volume at the out flowing profile,
\[ p(W_{ip=T}) \] - Exceedance probability of flood-wave volume at the i-th inflowing profile,
\[ p(W_{ij,r}) \] - Exceedance probability of flood-wave volume at the j-th inflowing profile,
\[ p(W_{ik,z}) \] - Exceedance probability of flood-wave volume at the k-th inflowing profile.

These relationships can be set up for a set of arbitrarily chosen values of \( \theta \) at the downstream profile of the considered river reach. Therefore, for the known volumes of flood hydrograph at upstream (inflowing) profiles, using Eq. (17) and Eq. (20), the flood volume at the out flowing profile can be calculated.

**PRACTICAL APPLICATION**

The developed methodology for evaluation of multiple coincidences of flood waves at main river and its tributary was applied to a sector of the Danube River in Serbia, with two tributaries: the Tisza and Sava Rivers. In this particular case, probabilities of multidimensional coincidence of flood events on the Danube River was evaluated for the gauging station Pancevo (downstream from Belgrade), as a function probabilities associated with the flood waves at the inflowing profiles: GS Bogojevo (the Danube River inflowing profile), GS Senta and GS Sremska Mitrovica (at the tributaries – The Tisza and Sava Rivers respectively). A schematic outline of the considered sector is given in Fig. 1.

![Figure 1. A schematic presentation of the Danube River sector for analyses of flood coincidence](image-url)
Flood volume was selected as referent variable for analysis. Due to pronounced complexity of flood origin on the considered rivers, there were analyzed the largest annual flood-volumes above discharge corresponding to 50% duration of the daily flow. It was assumed that flood wave of the Danube River at GS Pancevo with the exceeding probability, $\theta$, is formed by corresponding floods at Bogojevo having the exceeding probability $p$ (return period $T$) and arbitrarily chosen combinations two-dimensional coincidence of flood at the Tisza River near Senta and the Sava River near Sremska Mitrovica. Using the devised procedure there was formed a relationship of the probabilities of coincidence of flood events at the considered stations in the form:

$$p(W_{iz,p=\theta}^{Pan}) = P((p(W_{i,p=T}^{Bog}) \cap p(W_{j,r}^{Sen}) \cap p(W_{k,z}^{Sr.M}))$$  \hspace{1cm} (21)

where:

- $p(W_{iz,p=\theta}^{Pan})$ – Exceeding probability of flood volume of the Danube River at exiting profile near Pancevo,
- $p(W_{i,p=T}^{Bog})$ – Exceedance probability of flood volume at the inflowing profile of the Danube River at GS Bogojevo,
- $p(W_{j,r}^{Sen})$ – Probability of flood wave of the Tisza River near Senta (inflow profile into the considered river section),
- $p(W_{k,z}^{Sr.M})$ – Probability of flood wave of the Sava River near Sremska Mitrovica (inflow profile into the considered river section).

The results of evaluation of the multiple coincidence of floods on the Danube, Tisza and Sava Rivers for a 100-year flood of the Danube River near Pancevo are presented in Figure 2.
Figure 2. Multiple coincidence of 100-year flood of the Danube River near Pancevo as a function of floods of the Danube River near Bogojevo and the Tisza and Sava Rivers near Senta and Sremska Mitrovica, respectively

PRACTICAL IMPORTANCE OF THE RESULTS

The concrete results of multiple coincidences of floods on the Danube River and its tributaries have manifold significance:
- Provide for an assessment of the flood genesis pattern on the Danube River and tributaries;
- Give a foundation for the development of line-systems for flood protection within mutual interaction of main river and tributaries;
- Render assistance in setting up a strategy for development and improvement of flood protection systems in riverside within confluence of significant river flows;
- Represent hydrological data base for an assessment of statistical significance of the Danube River floods in Serbia;
- Can serve as support in making real time hydrologic forecasts an flood warnings;
- Offer tools for the flood risk assessment;

CONCLUSIONS

Numerous list of practical uses of the developed methodology for analyses of flood coincidence at the Danube River and its tributary the Tisza and Sava Rivers suggest broad opportunities for application in water resources management. The most frequent...
applicability is tied to flood protection and river bed training. It can, also be used in real time forecasting and flood warning. This paper contributes to an improvement of practical – engineering hydrology, particularly with regard to assessment of flood events and comprehensive flood-risk analyses.

References


